

Greg Call Daily Notes

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Types of Graphs

Thm Eigenvalues of (Δ) are non-negative (week long HW)

Types of Graphs

- **1. Complete Graphs:** A complete graph of order n is denoted by K_n .
- **2. Empty Graphs:** The empty graph on n vertices, denoted by E_n , is the graph of order n where E is the empty set.
- **3. Complements:** Given a graph G , the *complement* of G , denoted by \overline{G} , is the graph whose vertex set is the same as that of G , and whose edge set consists of all the edges that are not present in G .
- **4. Regular Graphs:** A graph G is regular if every vertex has the same degree. G is regular of degree r (i.e., r -regular) if $\deg(v) = r$ for all vertices v in G . Complete graphs of order n are regular of degree $n - 1$, and empty graphs are regular of degree 0.
- **5. Cycles:** The graph C_n is a cycle on n vertices.
- **6. Paths:** The graph P_n is a path on n vertices.
- **7. Subgraphs:** A graph H is a subgraph of a graph G if $V(H) \subset V(G)$ and $E(H) \subset E(G)$.
- **8. Induced Subgraphs:** Given a graph G and a subset S of the vertex set, the subgraph of G induced by S , denoted $\langle S \rangle$, is the subgraph with vertex set S and edge set $\{uv \mid u, v \in S \text{ and } uv \in E(G)\}$. So, $\langle S \rangle$ contains all the vertices of S and all edges of G whose end vertices are both in S .
- **9. Bipartite Graphs:** A graph G is bipartite if its vertex set can be partitioned into two sets X and Y in such a way that every edge of G has one end vertex in X and the other in Y . In this case, X and Y are called the *partite sets*. A bipartite graph with partite sets X and Y is called a complete bipartite graph if its edge set is of the form $E = \{xy \mid x \in X, y \in Y\}$ (that is, if every possible connection of a vertex in X with a vertex in Y is present). Such a graph is denoted by $K_{|X|,|Y|}$.

- **10. Tree Graphs:** Connected graph with no cycles
- **11. Star Graphs:** A tree with exactly one vertex of degree greater than 1 (the central vertex) and all other vertices having degree 1 (leaf vertices)